Azonosító jel:

ÉRETTSÉGI VIZSGA • 2019. május 7.

MATEMATIKA ANGOL NYELVEN

EMELT SZINTŰ ÍRÁSBELI VIZSGA

2019. május 7. 8:00

Időtartam: 300 perc

Pótlapok száma	
Tisztázati	
Piszkozati	

EMBERI ERŐFORRÁSOK MINISZTÉRIUMA

Instructions to candidates

1. The time allowed for this examination paper is 300 minutes. When that time is up, you will have to stop working.

2. You may solve the problems in any order.

3. In part II, you are only required to solve four of the five problems. **When you have finished the examination, enter the number of the problem not selected in the square below**. *If it is not clear* for the examiner which problem you do not want to be assessed, the last problem in this examination paper will not be assessed.

4. On solving the problems, you may use a calculator that cannot store and display textual information. You may also use any edition of the four-digit data tables. The use of any other electronic device or printed or written material is forbidden!

5. **Always write down the reasoning used to obtain the answers. A major part of the score will be awarded for this.**

6. **Make sure that calculations of intermediate results are also possible to follow.**

7. **The use of calculators** in the reasoning behind a particular solution **may be accepted without further mathematical explanation in case of the following operations**: addition, subtraction, multiplication, division, calculating powers and roots, $n!$, $\binom{n}{k}$, replacing the tables found in the 4-digit Data Booklet (sin, cos, tan, log, and their inverse functions), approximate values of the numbers π and e, finding the solutions of the standard quadratic equation. No further explanation is needed when the calculator is used to find the mean and the standard deviation, as long as the text of the question does not explicitly require the candidate to show detailed work. **In any other cases, results obtained through the use of a calculator are considered as unexplained and points for such results will not be awarded.**

8. On solving the problems, theorems studied and given a name in class (e.g. the Pythagorean Theorem or the height theorem) do not need to be stated precisely. It is enough to refer to them by name, but their applicability needs to be briefly explained. Reference to other theorems will be fully accepted only if the theorem and all its conditions are stated correctly (proof is not required) and the applicability of the theorem to the given problem is explained.

9. Always state the final result (the answer to the question of the problem) in words, too!

10. Write in pen. Diagrams may be drawn in pencil. The examiner is instructed not to mark anything written in pencil, other than diagrams. If you cancel any solution or part of a solution by crossing it over, it will not be assessed.

11. Only one solution to each problem will be assessed. In case of more than one attempt to solve a problem, **indicate clearly** which attempt you wish to be marked.

12. Please, **do not write in the grey rectangles**.

I.

1. Solve the following equations in the set of real numbers.

a) $2^{2x+2}+31\cdot 2^{x}-8=0$

b) $4\sin^{3}x-\sin x=0$

a)	6 points	
b)	7 points	
T.:	13 points	

2. While planning a minimal-cost cable network connecting several towns, a **complete graph** was first drawn. In this graph each town was represented by a vertex and each cable connection was represented by an edge. The edges had also been labelled to show how much that particular connection would cost to build. Then the "expensive edges" were deleted, one after the other, so that the resulting graph would still remain connected. After deleting two thirds of all edges of the complete graph, the remaining **tree graph** represented the minimal-cost network.

a) How many towns are to be connected by this network?

Ten towns entered their soccer teams into the fall season tournament, one team per town. Each team played one game with every other team. The winner of each game received 3 points, the losing team received 0 points and both teams received 1 point each in case of a draw. At the end of the tournament the ten teams had a total of 130 points.

b) How many games ended with a draw?

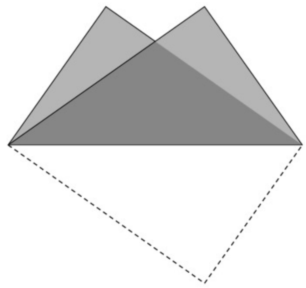
a)	6 points	
b)	5 points	
T.:	11 points	

3. Consider all seven-digit natural numbers that contain all of the digits 1, 2, 3, 4, 5, 6, 7.

a) These seven-digit numbers are written on 0.5 cm × 2 cm rectangular slips of paper (one number per slip). Will 8 sheets of A4 paper be enough to make all the required slips? (An A4 sheet is rectangular and it measures 21 cm × 29.7 cm.)

b) The above seven-digit numbers are arranged in increasing order.
Prove that the 721st number will be 2 134 567.

A 21 cm × 29.7 cm (A4) sheet is folded along one of its diagonals (see diagram).



c) Calculate the area of the overlap of the two folded halves.

a)	4 points	
b)	4 points	
c)	5 points	
T.:	13 points	

4. Four sensors (A, B, C, D) are planted on the horizontal seafloor near the shore. The positions of three of these sensors are given on the plans in a rectangular coordinate system as $A(0; –12.5)$, $B(10; –7.5)$, $C(48; 14)$.

a) Prove that points A, B and C are not collinear (i.e. they are not on the same line).

Each unit on the axes of the coordinate system on the plan represents a distance of 20 metres.

b) How many metres is the real distance between sensors A and D if sensor D is planted so that it is equally far from sensors A and B and exactly 1000 metres from sensor C?

a)	4 points	
b)	10 points	
T.:	14 points	

II.

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 2.

5. A fair gambling die is rolled twice. The first number shown will be considered as the first term of an arithmetic sequence, while the second number will be the common difference of this sequence.

a) For how many of the above sequences would the sum of the first 10 terms be less than 100? (Two sequences are considered different if their first terms or their common differences are different.)

Consider all four-digit positive integers that do not have 0 among their digits.

b) In some of these numbers the four digits form four consecutive terms of an arithmetic sequence (the order of the digits may be changed). How many such numbers are possible?

Janka has already rolled a fair die four times. If she got a 3 on her fifth roll, the mean of her five rolls would also be 3. If she rolled a 4 for the fifth time, the median of her five rolls would also be 4. If she rolled a 5 for the fifth time, the (single) mode of her five rolls would also be 5.

c) What numbers could Janka have rolled for the first four times? (Ignore the order of these numbers.)

a)	5 points	
b)	5 points	
c)	6 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 2.

6. The length of the bounding arc of a radius r sector is i, the perimeter is $2r + i = 10$ cm.

a) Let the radius of the sector be 2 cm. Determine the central angle α and the area A of the sector, as well as the radius R of the base circle of the straight cone whose lateral surface is this sector.

b) Consider all sectors of perimeter 10 cm. Prove that the sector that has the maximal area has a central angle of 2 radians.

c) Determine whether the following statement is true or false. Explain your answer. The area of a sector with a perimeter of 10 cm is always less than the area of a sector with a perimeter of 20 cm.

a)	5 points	
b)	8 points	
c)	3 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 2.

7. There are 4 red and 3 green marbles in a box. Another s yellow marbles are also placed in the box. Two of the marbles are then selected **with replacement**.

a) Determine the value of s if the probability that both marbles are green is 0.09.

There are 4 red, 3 green and k blue marbles in a box ($k \geq 1$). Three marbles are selected from the box **without replacement**.

b) Prove that the probability that all three marbles will be of a different colour is $\frac{72k}{(k+7)(k+6)(k+5)}$.

c) Determine the value of k if the probability of selecting three marbles of different colours is equal to the probability of selecting three blue marbles.

a)	4 points	
b)	5 points	
c)	7 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 2.

8. The surface of Lake Balaton is about 76.5 km long and has an average width of about 7.7 km.

a) Calculate the average depth of Lake Balaton, given that the estimated volume of the water in the lake is 2 billion m^3. Give your answer in metres, rounded to one decimal place.

Ádám and Misi are planning to ride around the lake by bicycle in a single day. The bicycle route around the lake is 205 km long. They start off at 7 a.m. When they stop for lunch they find that their average speed that far has been 16 km/h. After a 60 minute lunch break they start off again. To complete the journey before nightfall they increase their average speed to 20 km/h for the rest of the day, arriving back to their starting point by 7:30 p.m.

b) When did the boys have their lunch break?

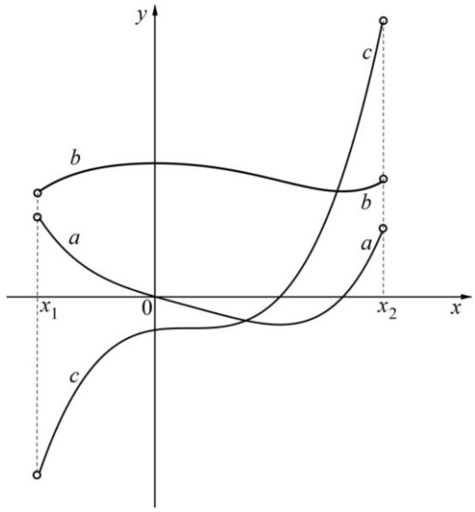
The lake is widest between Balatonvilágos and Balatonalmádi, about 12.7 km.

c) Taking the curvature of Earth’s surface into account, at least how tall above the water level should the signpost in Balatonvilágos harbour rise, so that storm warning light fixed at its top could still be seen by people bathing at the Balatonalmádi lakeshore? (Consider the Earth a sphere of radius 6370 km.)

a)	3 points	
b)	6 points	
c)	7 points	
T.:	16 points	

You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 2.

9. Function f is defined over the open interval $]x_1; x_2[$. The diagram shows the graph of function f as well as the graphs of its first and second derivatives. The three graphs, not necessarily in this order, are labelled a, b and c.
According to statement **A** in the table below, a is the graph of function f, b is the graph of its first derivative (f') and c is the graph of its second derivative (f'').
All the other possibilities are similarly listed.



a) Determine the truth value of statements **B, C, D, E, F**. No further explanation is required **here**.
(Statement **A** is already given as *false*.)

	f	f'	f''	the statement is true/false
A	a	b	c	false
B	a	c	b	
C	b	a	c	
D	b	c	a	
E	c	a	b	
F	c	b	a	

b) Based on the relations between a function and its derivatives explain why statement **A** must be false.

Points A, B, C, D are given in a rectangular coordinate system: $A(0; 4)$, $B(0; 1)$, $C(p; 1)$, $D(p; 4)$, where $p > 0$. The graph $y = \frac{x^2}{4}$ bisects the area of rectangle $ABCD$.

c) Prove that $p > 4$ and also calculate the exact value of p.

a)	3 points	
b)	3 points	
c)	10 points	
T.:	16 points	

	Number of problem	score			
		maximum	awarded	maximum	awarded
Part I	1.	13		**51**	
	2.	11			
	3.	13			
	4.	14			
Part II		16		**64**	
		16			
		16			
		16			
		← problem not selected			
Total score on written examination				**115**	

date

examiner

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	pontszáma **egész számra** kerekítve	
	elért	programba beírt
I. rész		
II. rész		

dátum

dátum

javító tanár

jegyző